Published by Institute of Physics Publishing for SISSA

Received: *December 6, 2006*
Revised: *February 23, 2007*
Accepted: *February 26, 2007*
Published: *March 12, 2007*

Boundary states, matrix factorisations and correlation functions for the E-models

Christoph A. Keller and Sebastiano Rossi

Institut für Theoretische Physik, ETH Zürich
CH-8093 Zürich, Switzerland
E-mail: kellerc@itp.phys.ethz.ch, rossise@itp.phys.ethz.ch

Abstract: The open string spectra of the B-type D-branes of the $N = 2$ E-models are calculated. Using these results we match the boundary states to the matrix factorisations of the corresponding Landau-Ginzburg models. The identification allows us to calculate specific terms in the effective brane superpotential of E_6 using conformal field theory methods, thereby enabling us to test results recently obtained in this context.

Keywords: D-branes, Conformal Field Models in String Theory, Topological Strings.

 http://jhep.sissa.it/archive/papers/jhep032007038/jhep032007038.pdf

Contents

JHEP03(2007)038

1. Introduction

$N = 2$ minimal models with an ADE classification play a central role in the description of certain Calabi-Yau compactifications. In particular, they form the building blocks of Gepner models [1]. For this reason, there has been great interest in the branes of these models and their spectrum.

In the language of abstract conformal field theory, branes are given by boundary states. They are linear combinations of Ishibashi states and must satisfy the Cardy condition. On the other hand, these models can also be described as Landau-Ginzburg models. In this case branes correspond to matrix factorisations of the superpotential [2 – 8]. An interesting problem is thus to compare these descriptions by matching boundary states to matrix factorisations. For the A- and D-models, this has been done in [3, 5] and [9], respectively.

In this paper, we perform the match for the $N = 2$ E-models. For these models the complete set of matrix factorisations has been known to mathematicians for some time [10, 11]. On the CFT side, the boundary states have been constructed in [12 – 14]. We calculate their spectrum and match the two descriptions.

We then use the identification to discuss obstructions to brane deformations. The critical loci of the effective superpotential W_{eff} describe the directions in which a given matrix factorisation can be deformed, and nonvanishing potential terms describe obstructions to deformations [15, 16]. On the other hand, W_{eff} is also the generating functional of open string topological disk correlators [17]. Using our identification, we show that certain specific correlators do not vanish, so that the brane deformation in these directions is obstructed. This calculation can then be used to test results obtained using other approaches [18].

This paper is organised as follows: In section 2, we recall the ADE classification for affine $su(2)$ models and the construction of their boundary states. For later use we list some basic properties of $N = 2$ minimal models, the exceptional Lie groups E_n, and matrix factorisations. In section 3, for each model and each choice of GSO-projection, we first assemble all information on matrix factorisations and boundary states. We then calculate their spectrum and match the boundary states to their corresponding matrix factorisations. In section 4, we use this identification to calculate topological correlators to get certain specific terms of the effective superpotential. We then draw our conclusions in section 5.

2. Basics

2.1 Matrix factorisations

The topological part of an $N = 2$ minimal model can also be described in terms of a Landau-Ginzburg model. The superpotential W is a weighted homogeneous polynomial in x_i. For E_n, the superpotentials and the charges q_i of the variables are given in table 1. Note that for each model there are two different superpotentials which correspond to the two choices of GSO-projections [5]: the two variable potentials give type $0B$ projection, the three variable potentials type $0A$.

JHEP03(2007)038

	h	$\mathcal{E}$	GSO	q_i
E_6	12	1,4,5,7,8,11	$W = x^3 + y^4$ (0B) $W = x^3 + y^4 + z^2$ (0A)	$[x] = \frac{2}{3},\ [y] = \frac{1}{2},\ [z] = 1$
E_7	18	1,5,7,9,11,13,17	$W = x^3 + xy^3$ (0B) $W = x^3 + xy^3 + z^2$ (0A)	$[x] = \frac{2}{3},\ [y] = \frac{4}{9},\ [z] = 1$
E_8	30	1,7,11,13,17,19,23,29	$W = x^3 + y^5$ (0B) $W = x^3 + y^5 + z^2$ (0A)	$[x] = \frac{2}{3},\ [y] = \frac{2}{5},\ [z] = 1$

Table 1: Exceptional groups and their superpotential

B-type branes in the Landau-Ginzburg description are given by square matrices E, J with polynomial entries, and a charge matrix R. E, J satisfy

$$E\,J = J\,E = W\,\mathbf{1}\,, \tag{2.1}$$

or equivalently,

$$Q^2 = W\,\mathbf{1} \qquad \text{where } Q = \begin{pmatrix} 0 & J \\ E & 0 \end{pmatrix}\ . \tag{2.2}$$

In our conventions W has $\mathrm{U}(1)$ charge 2 and Q has charge 1:

$$e^{i\lambda R}\,Q(e^{i\lambda q_i}x_i)\,e^{-i\lambda R} = e^{i\lambda}Q(x_i)\ . \tag{2.3}$$

To determine R uniquely, one must in addition fix $\operatorname{tr} R$ (see [11] for details).

Define the operator D by

$$D(\phi) := Q_2\phi - (-1)^{\deg(\phi)}\phi\,Q_1\,, \tag{2.4}$$

where $\deg(\phi)$ is the natural $\mathbb{Z}_2$-grading of ϕ: even for bosons, odd for fermions. The topological spectrum between Q_1, Q_2 is given by morphisms $\phi(x_i)$ in the cohomology of D. The charge q of ϕ is given by

$$e^{i\lambda R_2}\,\phi(e^{i\lambda q_i}x_i)\,e^{-i\lambda R_1} = e^{i\lambda q}\,\phi(x_i)\ . \tag{2.5}$$

The antibrane $\bar{Q}$ of Q is obtained by interchanging E and J. Note that the even spectrum between two branes is equivalent to the odd spectrum between brane and antibrane and vice versa.

2.2 The affine SU(2) case

In this subsection we start the CFT description of $N = 2$ minimal models. In view of their construction as cosets (see 2.3) we will first consider $su(2)$ models. The ADE classification gives all possible modular invariant partition functions obtained from combinations of $su(2)_k$ characters. Each such partition function corresponds to a simply laced Lie algebra A_n, D_n, or E_n.

JHEP03(2007)038

Here we are interested only in the exceptional groups E_n. Their Dynkin diagrams and other properties can be found in tables 1 and 2. The corresponding partition functions are given by:

$$\begin{aligned}
Z_{E_6} &= |\chi_0+\chi_6|^2+|\chi_3+\chi_7|^2+|\chi_4+\chi_{10}|^2 && (k=10)\\
Z_{E_7} &= |\chi_0+\chi_{16}|^2+|\chi_4+\chi_{12}|^2+|\chi_6+\chi_{10}|^2+|\chi_8|^2 && \\
&\quad +\chi_8(\bar\chi_2+\bar\chi_{14})+(\chi_2+\chi_{14})\bar\chi_8 && (k=16)\\
Z_{E_8} &= |\chi_0+\chi_{10}+\chi_{18}+\chi_{28}|^2+|\chi_6+\chi_{12}+\chi_{16}+\chi_{22}|^2 && (k=28)
\end{aligned}$$

where the χ_λ are $su(2)_k$ characters, and k is related to the Coxeter number h of E_n by $h=k+2$. The boundary states of these models have been constructed some time ago [12]: To each node L of the Dynkin diagram there corresponds a boundary state given by

$$||L\rangle\rangle = \sum_{l+1\in\mathcal{E}} \frac{\psi_L^{(l)}}{\sqrt{S_0^l}}|[l]\rangle\rangle\ . \tag{2.6}$$

Here $l+1$ runs over the Coxeter exponents of E_n. The $\psi_L^{(l)}$ for each model are listed in appendix B. The modular transformation matrix is

$$S_L^l = \sqrt{\frac{2}{h}}\sin\left(\pi\frac{(L+1)(l+1)}{h}\right)\ . \tag{2.7}$$

The overlap of two boundary states is then given by

$$\langle\langle L_1||q^{(L_0+\bar L_0)/2-c/24}||L_2\rangle\rangle = \sum_{l=0}^{k}\chi_l(\tilde q)\, n_{lL_1}^{\ \ L_2}\ . \tag{2.8}$$

The matrices $(n_i)_{L_1}^{L_2}$ are the so-called fused adjacency matrices [12]. They can be obtained recursively by applying $su(2)_k$ fusion rules

$$n_{i+1} = n_1 n_i - n_{i-1}\,,\quad i\le k-1, \tag{2.9}$$

where n_0 is the identity matrix and n_1 is the adjacency matrix of the Dynkin diagram. By construction the n_i form an integer valued representation of the fusion algebra, and explicit calculation shows that they are non-negative as well. The $||L\rangle\rangle$ thus satisfy the Cardy condition.

2.3 The $N=2$ minimal model

We consider now $N=2$ minimal models. Their bosonic subalgebra can be described as the coset

$$\frac{su(2)_k\oplus u(1)_4}{u(1)_{2k+4}}\ . \tag{2.10}$$

The representations of the coset are labelled by triples (l,m,s), where $l=0,\ldots,k$ is twice the spin of $su(2)$, $m\in\mathbb{Z}_{2k+4}$, and $s\in\mathbb{Z}_4$. The representations must obey $l+m+s=0$ mod 2 and are subject to the identification

$$(l,m,s)\sim(k-l,m+k+2,s+2)\ . \tag{2.11}$$

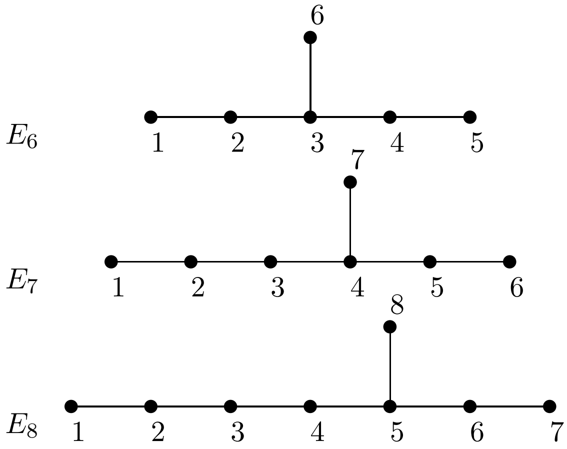

Table 2: Dynkin diagrams of the exceptional groups.

The conformal weights and U(1) charges of the highest weight states are up to integers given by

$$h(l,m,s) = \frac{l(l+2)-m^2}{4(k+2)} + \frac{s^2}{8}\,, \tag{2.12}$$

$$q(l,m,s) = \frac{s}{2} - \frac{m}{k+2}\,. \tag{2.13}$$

In the NS sector (s even), the chiral primaries appear in the representations $(l,l,0)$. In the R sector (s odd), the R ground states appear in $(l,l+1,1)$.

The characters $\chi_{[l,m,s]}(q)$ transform under the modular S-transformation as

$$\chi_{[L,M,S]}(q) = \sum_{[l,m,s]} S^{lms}_{LMS}\,\chi_{[l,m,s]}(\tilde{q})\,, \tag{2.14}$$

where the sum is over distinct equivalence classes. The S-matrix is given by

$$S^{lms}_{LMS} = \frac{1}{\sqrt{2h}}\,S^l_L\,e^{\frac{i\pi}{h}mM}e^{-\frac{i\pi}{2}sS}, \tag{2.15}$$

where S^l_L is the S-matrix of $su(2)$ (2.7). Let

$$Z = \sum_{l,\bar{l}} A_{l,\bar{l}}\chi_l\bar{\chi}_{\bar{l}} \tag{2.16}$$

be an ADE-modular invariant of $su(2)$. Then we can construct two different $N=2$ modular invariants by [19]

$$Z = \sum A_{l,\bar{l}}\chi_{[l,m,s]}\bar{\chi}_{[\bar{l},m,\pm s]}\,. \tag{2.17}$$

Physically, the choice $s=\bar{s}$ corresponds to type 0B GSO-projection, and $s=-\bar{s}$ to type 0A. See [20] for the complete list of all possible modular invariants of $N=2$ superconformal minimal models.

We want to construct boundary states $||B\rangle\rangle$ that satisfy B-type gluing conditions

$$
\begin{aligned}
(L_n - \bar{L}_{-n})||B\rangle\rangle &= 0\,, \\
(J_n + \bar{J}_{-n})||B\rangle\rangle &= 0\,, \\
(G_r^{\pm} + i\eta\,\bar{G}_{-r}^{\pm})||B\rangle\rangle &= 0\,,
\end{aligned}
\tag{2.18}
$$

where $\eta = \pm 1$ determines the spin structure. The boundary states of the E-models are then given by [13]

$$
||L, M, S\rangle\rangle = K \sum_{[l,m,s]} \frac{\psi_L^{(l)}}{\sqrt{S_{000}^{lms}}} e^{\frac{i\pi}{h}Mm} e^{-\frac{i\pi}{2}sS} |[l,m,s]\rangle\rangle, \tag{2.19}
$$

where h is the Coxeter number of the group and $\psi_L^{(l)}$ are the coefficients of the corresponding $su(2)$ model. The overall normalisation K depends on the model and the type of GSO-projection.

The Ishibashi states $|[l,m,s]\rangle\rangle$ live in sectors with $m = -\bar{m}$ and $s = -\bar{s}$, and the sum in (2.19) is over distinct equivalence classes. $||L, M, S\rangle\rangle$ satisfies (2.18) with $\eta = 1$ ($\eta = -1$) for S even (S odd). In section 3 we will discuss the exact ranges of l, m, s and L, M, S for each case individually.

The chiral primaries $(l, l, 0)$ in the overlap between two boundary states $||B_1\rangle\rangle$ and $||B_2\rangle\rangle$ should then correspond one-to-one to the morphisms in the cohomology between the two corresponding matrix factorisations Q_1, Q_2 — in particular, their U(1) charges given by (2.5) and (2.13) respectively, must be equal. By calculating and comparing the spectra, we can thus match matrix factorisations to boundary states.

3. The exceptional models: $E_6,\ E_7,\ E_8$

3.1 Branes of E_6

3.1.1 Type 0B: $W = x^3 + y^4$

This case corresponds to $m = \bar{m}$, $s = \bar{s}$ in (2.17). There are 12 Ishibashi states $|[l,m,s]\rangle\rangle$, $l + 1 \in \mathcal{E}(E_6)$, $l + m + s$ even, and $m = 0$ or 6 depending on the value of l:

$$
\begin{array}{llllll}
|[0,0,0]\rangle\rangle, & |[4,0,0]\rangle\rangle, & |[6,0,0]\rangle\rangle, & |[10,0,0]\rangle\rangle, & |[3,6,1]\rangle\rangle, & |[7,6,1]\rangle\rangle, \\
|[0,0,2]\rangle\rangle, & |[4,0,2]\rangle\rangle, & |[6,0,2]\rangle\rangle, & |[10,0,2]\rangle\rangle, & |[3,6,-1]\rangle\rangle, & |[7,6,-1]\rangle\rangle.
\end{array}
\tag{3.1}
$$

The boundary states are given by

$$
||L, M, S\rangle\rangle = \frac{1}{\sqrt{2}} \sum \frac{\psi_L^{(l)}}{\sqrt{S_{000}^{lms}}} e^{\frac{i\pi}{12}mM} e^{-\frac{i\pi}{2}sS}\, |[l,m,s]\rangle\rangle\ , \tag{3.2}
$$

where $L = 1, \ldots, 6$ and $S, M \in \mathbb{Z}_4$ with $L + M + S$ even, and the sum runs over the Ishibashi states (3.1).

The map $\tau : S \mapsto S+2$ maps branes to antibranes, as it changes the sign of the coupling to RR states. Note that in this case there is the symmetry

$$\begin{aligned} ||2,S\rangle\rangle &= \tau(||4,S\rangle\rangle),\ ||1,S\rangle\rangle = \tau(||5,S\rangle\rangle), \\ ||3,S\rangle\rangle &= \tau(||3,S\rangle\rangle),\ ||6,S\rangle\rangle = \tau(||6,S\rangle\rangle). \end{aligned} \tag{3.3}$$

Moreover, we have $||L,M,S\rangle\rangle = ||L,M+2,S+2\rangle\rangle$. M is thus fixed by demanding that $L+M+S$ be even, and by (3.3) we can restrict S to 0,1. This means that we are left with 12 different boundary states, 6 for each choice of spin structure. Their spectrum is

$$\begin{aligned} \langle\langle L_1,M_1,S_1||q^{(L_0+\bar{L}_0)/2-c/24}||L_2,M_2,S_2\rangle\rangle &= \frac{1}{2}\sum_{[l,m,s]}\chi_{[l,m,s]}(\tilde{q})\,\delta^{(2)}(S_1-S_2-s) \\ \times\left({n_{lL_2}}^{L_1}(1+e^{\frac{i\pi}{2}(S_2-S_1+s+M_2-M_1+m)}) + {n_{10-l\,L_2}}^{L_1}(1-e^{\frac{i\pi}{2}(S_2-S_1+s+M_2-M_1+m)})\right)&, \end{aligned} \tag{3.4}$$

where ${n_{lL_2}}^{L_1}$ are the fused adjacency matrices for E_6.

There are six matrix factorisations for this model, listed in appendix C.1. Their spectrum has been calculated in [18]. It agrees with the chiral primary fields of (3.4) if we make the identifications:

$$Q_L \equiv ||L,M,0\rangle\rangle \tag{3.5}$$

with $M\in\{0,1\}$ such that $L+M$ even for the spin structure $S=0$, and

$$Q_L \equiv ||L,M,1\rangle\rangle \tag{3.6}$$

with $M\in\{1,2\}$ such that $L+M$ odd for $S=1$.

3.1.2 Type 0A: $W = x^3+y^4+z^2$

There are 12 Ishibashi states

$$|[l,0,s]\rangle\rangle \qquad l+1\in\mathcal{E}(E_6)\,, \tag{3.7}$$

with $s\in\mathbb{Z}_4$ such that $l+s$ even. The boundary states are given by

$$||L,S\rangle\rangle = ||L,0,S\rangle\rangle = \frac{1}{\sqrt{2}}\sum\frac{\psi_L^{(l)}}{\sqrt{S_{000}^{lms}}}\,e^{-\frac{i\pi}{2}sS}\,|[l,m,s]\rangle\rangle\,, \tag{3.8}$$

the sum running over the Ishibashi states (3.7). We have $L=1,\ldots,6$ and $S\in\mathbb{Z}_4$, but again the symmetry under τ allows us to restrict $S\in\{0,1\}$, so that we have 6 boundary states per spin structure.

Their overlap is

$$\begin{aligned} &\langle\langle L_1,S_1||q^{(L_0+\bar{L}_0)/2-c/24}||L_2,S_2\rangle\rangle = \\ &\qquad \sum_{[l,m,s]}\chi_{[l,m,s]}(\tilde{q})\left({n_{lL_2}}^{L_1}\delta^{(4)}(S_1-S_2-s)+{n_{10-l\,L_2}}^{L_1}\delta^{(4)}(S_1-S_2+2-s)\right)\,. \end{aligned} \tag{3.9}$$

The matrix factorisations of $W=x^3+y^4+z^2$ are listed in appendix C.1, and their spectrum has been calculated in [11] (beware of the difference in labelling!) It agrees with (3.9) if we identify

$$Q_L \equiv ||L,S\rangle\rangle\,. \tag{3.10}$$

JHEP03(2007)038

3.2 Switching between GSO-projections

3.1.1 and 3.1.2 illustrate nicely how one can change between one GSO-Projection and the other: One constructs the new branes out of the old branes by orbifolding by τ. For instance, if we start out with the type 0A theory, we take the orbits of all branes that are not invariant,

$$\begin{aligned}||3, M, S\rangle\rangle &= \frac{1}{\sqrt{2}}(||2, S\rangle\rangle + ||4, S\rangle\rangle)\,,\\ ||6, M, S\rangle\rangle &= \frac{1}{\sqrt{2}}(||1, S\rangle\rangle + ||5, S\rangle\rangle)\,.\end{aligned}$$

We have thus projected out the Ramond part of these branes.

On the other hand, a fixed point $||B\rangle\rangle$ of τ corresponds to a fractional brane which must be resolved by adding linear combinations of the new Ramond Ishibashi states, i.e.

$$\begin{aligned}||B_1\rangle\rangle &= \frac{1}{\sqrt{2}}||B\rangle\rangle + \text{linear combination of new states}\,,\\ ||B_2\rangle\rangle &= \frac{1}{\sqrt{2}}||B\rangle\rangle - \text{linear combination of new states}\,.\end{aligned}$$

It can be checked that by this procedure we really obtain the boundary states (3.2) of the type 0B theory.

3.3 Branes of E_7

3.3.1 Type 0B: $W = x^3 + xy^3$

E_7 is insofar different from E_6 as the two GSO-projections have a different number of boundary states. For type 0B projection, there are 28 Ishibashi states,

$$|[l, 0, s]\rangle\rangle \qquad l + 1 \in \mathcal{E}(E_7),\ s \in \{0, 2\}\,, \tag{3.11}$$

and

$$|[l, 9, s]\rangle\rangle \qquad l + 1 \in \mathcal{E}(E_7),\ s \in \{-1, 1\}\ . \tag{3.12}$$

The boundary states are

$$||L, M, S\rangle\rangle = \frac{1}{2}\sum_{\substack{l+1\in\mathcal{E},\ m=0,9\\ m+s\ \text{even}}} \frac{\psi_L^{(l)}}{\sqrt{S_{000}^{lms}}} e^{\frac{i\pi}{18}mM} e^{-\frac{i\pi}{2}sS} |[l, m, s]\rangle\rangle\,, \tag{3.13}$$

where $L = 1, \ldots 7$, $S = 0, 1, 2, 3$ with $L+M+S$ even. This time the $\psi_L^{(l)}$ are the coefficients for the affine E_7 model given in appendix B.2. Again, S odd and S even give two different spin structures with 14 boundary states each.

The overlap is

$$\begin{aligned}&\langle\langle L_1, M_1, S_1 || q^{(L_0+\bar{L}_0)/2-c/24} || L_2, M_2, S_2\rangle\rangle = \\ &\qquad \frac{1}{2}\sum_{[l,m,s]} \chi_{[l,m,s]}(\tilde{q})\, n_{lL_1}{}^{L_2} \delta^{(2)}(S_1 - S_2 - s)\left(1 + e^{\frac{i\pi}{2}(M_2+S_2-M_1-S_1+m+s)}\right)\,,\end{aligned} \tag{3.14}$$

where the ${n_{lL_1}}^{L_2}$ are now the fused adjacency matrices for E_7.

The matrix factorisations are given in appendix C.2. Their spectrum agrees with (3.14) if we make the identification

$$Q_L \equiv ||L, M, 0\rangle\rangle\,, \;\; \bar{Q}_L \equiv ||L, M, 2\rangle\rangle\,, \tag{3.15}$$

with $M \in \{0, 1\}$ such that $L + M$ even, and

$$Q_L \equiv ||L, M, 1\rangle\rangle\,, \;\; \bar{Q}_L \equiv ||L, M, 3\rangle\rangle\,, \tag{3.16}$$

with $M \in \{1, 2\}$ such that $L + M$ odd.

3.3.2 Type 0A: $W = x^3 + xy^3 + z^2$

In this case we only have 14 Ishibashi states,

$$|[l, 0, s]\rangle\rangle \qquad l + 1 \in \mathcal{E}(E_7),\;\; s \in \{0, 2\}\;. \tag{3.17}$$

For the type 0B case, the map $\tau : S \mapsto S+2$ had no fixed points. It is thus straightforward to construct the boundary states for the 0A projection by

$$||L, S\rangle\rangle = \frac{1}{\sqrt{2}}(||L, M, S\rangle\rangle + ||L, M, S + 2\rangle\rangle)\;. \tag{3.18}$$

This gives the required 14 states. We could also have obtained these boundary states by using (2.19) with $K = \frac{1}{\sqrt{2}}$.

The overlap is

$$\begin{aligned}&\langle\langle L_1, S_1|| q^{(L_0+\bar{L}_0)/2-c/24} ||L_2, S_2\rangle\rangle\\ &\qquad = \sum_{[l,m,s]} {n_{lL_1}}^{L_2} \Big(\delta^{(4)}(S_1 - S_2 - s) + \delta^{(4)}(S_1 - S_2 + 2 - s)\Big)\, \chi_{[l,m,s]}(\tilde{q})\;.\end{aligned} \tag{3.19}$$

The identification with the matrix factorisations of appendix C.2 is

$$\hat{Q}_L \equiv ||L, S\rangle\rangle\;. \tag{3.20}$$

3.4 Branes of E_8

3.4.1 Type 0B: $W = x^3 + y^5$

The E_8 model is completely analogous to the E_7 model. For the 0B projection there are 32 Ishibashi states

$$\begin{aligned}&|[l, 0, s]\rangle\rangle \qquad l + 1 \in \mathcal{E}(E_8),\;\; s \in \{0, 2\}\,,\\ &|[l, 9, s]\rangle\rangle \qquad l + 1 \in \mathcal{E}(E_8),\;\; s \in \{-1, 1\}\,,\end{aligned}$$

and 32 boundary states $||L, M, S\rangle\rangle$, $L = 1 \dots 8$, $S = 0, 1, 2, 3$, $M = 0, 1$, $L + M + S$ even, given by (2.19) with $K = \frac{1}{2}$. Their spectrum is identical to (3.14) with ${n_{lL_1}}^{L_2}$ replaced by the fused adjacency matrices of E_8. The identification with the matrix factorisations of appendix C.3 is

$$Q_L \equiv ||L, M, 0\rangle\rangle\,, \;\; \bar{Q}_L \equiv ||L, M, 2\rangle\rangle \tag{3.21}$$

and

$$Q_L \equiv ||L, M, 1\rangle\rangle\,, \;\; \bar{Q}_L \equiv ||L, M, 3\rangle\rangle\;, \tag{3.22}$$

with M as in (3.15) and (3.16).

JHEP03(2007)038

3.4.2 Type 0A: $W = x^3 + y^5 + z^2$

Again, we only have 16 Ishibashi states. The 16 boundary states are constructed just as in (3.18), their spectrum is as in (3.19) and they are identified with the matrix factorisations of appendix C.3 by

$$\hat{Q}_L \equiv ||L, S\rangle\rangle \ . \tag{3.23}$$

4. Correlators and the effective superpotential

4.1 Introduction and motivation

In this section we make use of the previous match between boundary states and matrix factorisations to calculate specific correlators of the E_6-model. In particular, we choose correlators which will play a role for the effective superpotential of branes in this theory (see section 4.5). There exist different methods to calculate correlators in Landau-Ginzburg models, in particular the Kapustin-Li formula given in [5]. This formula however only works if there are no integrated operators, i.e. only if there are no more than 3 boundary operators or 1 bulk and 1 boundary operator.

We show how one can get around this restriction and evaluate Landau-Ginzburg correlators with integrated insertions by calculating an example for the E_6-model, the correlator of one bulk field xy and two boundary fields ψ. This is achieved by using the Kapustin-Li formula whenever possible, but changing to the picture of pure CFT if we encounter integrated insertions. This change is possible as we can identify all boundary states and fields of the problem.

We first gather some facts that will prove useful later on.

4.2 Decomposition of E_6

The fact that $c_{k=10} = c_{k=1} + c_{k=2}$ suggests that we can decompose E_6 into the simpler models A_1 and A_2. In terms of the LG potential, this corresponds to the observation that $W = x^3 + y^4$ is the sum of two A-model potentials.

We first decompose $k = 10$ characters to identify the Ishibashi states of the E_6-model with the A_1 and A_2-model Ishibashi states (see appendix A). Since each representation contains at most one such state, the identification is unambiguous up to a phase. Next, note that the character decomposition shows that $A_1 \otimes A_2$ and E_6 are equivalent at least as bulk theories. Any product of A_1 and A_2 boundary states preserves the diagonal $N = 2$ supersymmetry and is thus a boundary state of the E_6. Note that the reverse is not true — we thus expect that certain E_6 boundary states are not factorisable. After taking the tensor product, we have to spin align the state. This corresponds to a $\mathbb{Z}_2$ orbifold which eliminates all products of the form $NS \otimes R$. In principle, new twisted states are introduced by this action, but they do not satisfy our boundary conditions and therefore cannot contribute.

For a given spin structure, there are two A_1 and three A_2 boundary states [9] . One might thus expect to get six factorisable E_6 states, but this is not the case. The spin alignment eliminates the difference between some of the products, and it turns out that we are left with exactly three factorisable E_6 states. This agrees with a different analysis: the character decomposition in A suggests that factorisable boundary states must couple with the same strength to $|[0,0,0]\rangle\rangle$ and $|[6,0,0]\rangle\rangle$. In other words, the absolute value of the respective coefficients $\psi_L^{(0)}(S_0^0)^{-1/2}$ and $\psi_L^{(6)}(S_0^6)^{-1/2}$ must be the same. This is only true for the boundary states with $L = 1, 5, 6$.

To make the actual identification it is then sufficient to write out the tensor product of the boundary states. The character decompositions then fix at least the absolute values of the coefficients. This is sufficient to make the identification

$$\begin{aligned} ||0,0\rangle\rangle_1 \otimes ||1,0\rangle\rangle_2 &\sim ||1,0\rangle\rangle_{E_6} , \\ ||0,0\rangle\rangle_1 \otimes ||1,2\rangle\rangle_2 &\sim ||5,0\rangle\rangle_{E_6} , \\ ||0,0\rangle\rangle_1 \otimes ||0,0\rangle\rangle_2 &\sim ||6,0\rangle\rangle_{E_6} . \end{aligned} \tag{4.1}$$

This can be seen on the Landau-Ginzburg side as well: Q_1, Q_5, and Q_6 are tensor products, all the other Q contain terms of the form xy and cannot be decomposed (see appendix C.1).

4.3 Topological correlators

To obtain a topological conformal field theory, one can twist an $N = 2$ superconformal model. The correlators of this topological theory then have a natural interpretation in the original $N = 2$ theory. On the sphere, by standard methods the topological correlator is obtained by first inserting into the original correlation function a spectral flow operator $\rho(\xi)$, multiplying by a factor $\xi^{c/3}$, and then taking the limit $\xi \to \infty$ [24].

The insertion of a spectral flow operator is motivated by charge considerations: After twisting, the theory has a U(1)-charge anomaly $\frac{-c}{3}$. This means that all correlators vanish unless their total charge is equal to $\frac{c}{3}$. We thus have to insert an operator with a charge that corresponds to this background charge. In [24], the operator inserted is one unit of bulk spectral flow, whereas here we insert a boundary spectral flow.

4.4 Calculating $\langle \phi_7 \psi_4 \int [G, \psi_4] \rangle$

We are now ready to calculate the correlator

$$\langle \phi_7 \, \psi_4 \int [G, \psi_4] \rangle \tag{4.2}$$

for the boundary condition $||1,0\rangle\rangle$. We have denoted the bulk fields ϕ_i and the boundary fields ψ_i by $k+2$ times their U(1) charge.

Using the results from previous sections, we can rewrite (4.2) as a Landau-Ginzburg model correlator. By the comments in section 4.2, Q_1 factorises as

$$Q_1 = \begin{pmatrix} 0 & x \\ x^2 & 0 \end{pmatrix} \odot \begin{pmatrix} 0 & y^2 - iz \\ y^2 + iz & 0 \end{pmatrix} , \tag{4.3}$$

where $\odot$ is the graded tensor product [8]. Its fermionic spectrum is

$$\psi := \begin{pmatrix} 0 & 1 \\ -x & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} , \; \psi_2 = y \, \psi \, .$$

Moreover, by comparing U(1) charges, we can identify

$$\begin{array}{c} \phi_7 \longleftrightarrow xy \\ \psi_4 \longleftrightarrow \psi \end{array} , \tag{4.4}$$

so that (4.2) becomes

$$\langle xy\,\psi \int dt\,(G^-_{-1/2}\psi)(t)\rangle \ . \tag{4.5}$$

This factorises as

$$\int dt\langle x \begin{pmatrix} 0 & 1 \\ -x & 0 \end{pmatrix} \left(G^-_{-1/2}\begin{pmatrix} 0 & 1 \\ -x & 0 \end{pmatrix}\right)(t)\rangle_{A_1}\,\langle\, y\,\mathbf{1}\,\mathbf{1}(t)\,\rangle_{A_2} + \\ \int dt\langle x \begin{pmatrix} 0 & 1 \\ -x & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -x & 0 \end{pmatrix}(t)\rangle_{A_1}\,\langle\, y\,\mathbf{1}\,(G^-_{-1/2}\mathbf{1})(t)\,\rangle_{A_2}\ . \tag{4.6}$$

In the second term, $\langle\cdots\rangle_{A_2}$ vanishes because its total charge is $\frac{1}{2}-1=-\frac{1}{2}$ instead of the required $\frac{c}{3}=\frac{1}{2}$. On the other hand, the A_2 correlator of the first term is independent of t. As it contains no integrated operator insertions, we can evaluate it using [5]:

$$\langle\, y\,\rangle_{A_2} = \frac{1}{2(2\pi i)^2}\oint dy\,dz\,\frac{y\cdot \mathrm{STr}(\partial_y Q\,\partial_z Q)}{\partial_y W_{A_2}\partial_z W_{A_2}} = \frac{i}{4}. \tag{4.7}$$

This correlator could of course also be evaluated using pure CFT methods, but one would have to be very careful about normalisation. We simply note that y corresponds to the field $\phi_{110}(z)\phi_{110}(\bar{z})$, and the boundary spectral flow to ψ_{2-20}. Thus their total charge vanishes and the fusion rules allow the correlator to be non zero, so that it could only vanish because of purely dynamical reasons.

For the A_1 correlator, on the other hand, we cannot use the formula of Kapustin-Li, as it contains an integrated insertion. We thus write it as a coset model CFT correlator. By comparing U(1) charges, we can identify the fields

$$\begin{array}{c} x \longleftrightarrow \phi_{110}(z)\phi_{110}(\bar{z})\,, \\ \begin{pmatrix} 0 & 1 \\ -x & 0 \end{pmatrix} \longleftrightarrow \psi_{110}(s)\ . \end{array}$$

Moreover we insert one unit of spectral flow $\psi_{1-10}(\xi)$. We thus have to calculate the correlator

$$\int dt\langle \phi_{110}(z)\,\phi_{110}(\bar{z})\,\psi_{112}(t)\psi_{110}(s)\,\psi_{1-10}(\xi)\rangle\,, \tag{4.8}$$

where we have used $G^-_{-1/2}\psi_{110}=\psi_{112}$. Our task is simplified further since the A_1-model is really just the free boson,

$$\frac{su(2)_1\oplus u(1)_2}{u(1)_3} = u(1)_6\,, \tag{4.9}$$

and we can identify (see e.g. [25])

$$\begin{array}{c} \phi_{110} \longleftrightarrow e^{\frac{i}{\sqrt{3}}X}\,, \\ \psi_{112} \longleftrightarrow e^{\frac{-i}{\sqrt{3}}2X}\,, \\ \psi_{1-10} \longleftrightarrow e^{\frac{-i}{\sqrt{3}}X}\ . \end{array}$$

Our original boundary state is a B-type brane and corresponds thus to Neumann boundary conditions for the free boson. We can use an explicit expression for (4.8) [23],

$$2\pi i\, C\, |z-\bar{z}|^{1/3}\, |z-s|^{2/3}\, |z-\xi|^{-2/3} |\xi-s|^{-1/3} \int dt\, |\xi-t|^{2/3}\, |s-t|^{-2/3}\, |z-t|^{-4/3}\,, \quad (4.10)$$

where C is a regularised functional determinant. To obtain the topological correlator, we have to multiply by $|\xi|^{1/3}$ and let $\xi \to \infty$. We can exchange limit and integral because for ξ large enough, the integrand is dominated by $(1+|t|^{2/3})|s-t|^{-2/3}|z-t|^{-4/3} \in L^1$. The result is then

$$\langle \cdots \rangle_{A_1} = 2\pi i\, C |z-\bar{z}|^{1/3} |z-s|^{2/3} \int \frac{dt}{|z-t|^{4/3}|s-t|^{2/3}} \neq 0\ . \quad (4.11)$$

We thus conclude that the correlator $\langle xy\, \psi \int dt\, (G^-_{-1/2}\psi)(t)\rangle$ does not vanish.

4.5 The effective brane superpotential

So far we have calculated a correlator which corresponds to a term in the generating functional of open string topological disk correlators. By standard lore (see e.g. [15–17]), the generating functional for symmetrised correlators is also the effective brane superpotential $W_{\rm eff}$. In particular, its critical loci give the directions in which a given brane can be deformed without being obstructed. There exist different methods to calculate $W_{\rm eff}$, two of which are presented in [18]. Firstly, using a generalised Massey product algorithm, one can determine the obstructed directions and from that deduce $W_{\rm eff}$. Secondly, the authors of [18] propose a 'mixed' approach. They apply the Massey product algorithm with all bulk insertions set to zero, and then combine these results with the generalized WDVV equations to obtain the general potential. The two methods give different results: In particular, $W_{\rm eff}^{\rm Massey}$ contains certain terms that $W_{\rm eff}^{\rm mixed}$ does not.

We can compare our result from 4.4 to the two effective superpotentials. In the notation of [18], the correlator we have calculated corresponds to the term $-\frac{1}{2}s_5u_4^2$. This term appears in $W_{\rm eff}^{\rm Massey}$, but not in $W_{\rm eff}^{\rm mixed}$. Moreover, all possible field redefinitions are constrained by R-charge compatibility. In our case this guarantees that $-\frac{1}{2}s_5u_4^2$ cannot be transformed away by such a field redefinition. A similar analysis shows that the correlator corresponding to the $W_{\rm eff}^{\rm Massey}$ term $s_8u_4u_1$ does not vanish either, again in disagreement with $W_{\rm eff}^{\rm mixed}$.

5. Conclusion

Our results for the exceptional models conclude the program started in [3, 5, 9]: For all ADE models, the match between matrix factorisations and boundary states is now known. We have also confirmed that the different GSO-projections correspond to superpotentials with and without additional z^2 terms.

The identification of matrix factorisations with boundary states allows one to calculate topological correlators using conformal field theory methods. In this paper we have demonstrated this for one of the correlators of the E_6-model. While in general this approach is

JHEP03(2007)038

likely to be complicated, there are cases (for example the correlator studied in this paper) where this is actually an efficient method. In any case, it allows one to check terms of the effective superpotential that characterise the obstructions of matrix factorisations under deformations.

Acknowledgments

This work has been partially supported by the Swiss National Science Foundation. We thank Ilka Brunner, Stefan Fredenhagen and Matthias Gaberdiel for their support and many helpful discussions. We thank Andreas Recknagel for helpful comments.

A. Character decompositions

The characters of the $N = 2$, $k = 10$ model can be decomposed into $k = 1$ and $k = 2$ characters in the following way:

$$\begin{aligned}
\chi^{(1)}_{[0,0,0]}\chi^{(2)}_{[0,0,0]} + \chi^{(1)}_{[0,0,2]}\chi^{(2)}_{[0,0,2]} &= \chi^{(10)}_{[0,0,0]} + \chi^{(10)}_{[6,0,0]} \\
\chi^{(1)}_{[0,0,2]}\chi^{(2)}_{[0,0,0]} + \chi^{(1)}_{[0,0,0]}\chi^{(2)}_{[0,0,2]} &= \chi^{(10)}_{[0,0,2]} + \chi^{(10)}_{[6,0,2]} \\
\chi^{(1)}_{[0,0,0]}\chi^{(2)}_{[2,0,2]} + \chi^{(1)}_{[0,0,2]}\chi^{(2)}_{[2,0,0]} &= \chi^{(10)}_{[4,0,0]} + \chi^{(10)}_{[10,0,2]} \\
\chi^{(1)}_{[0,0,0]}\chi^{(2)}_{[2,0,0]} + \chi^{(1)}_{[0,0,2]}\chi^{(2)}_{[2,0,2]} &= \chi^{(10)}_{[4,0,2]} + \chi^{(10)}_{[10,0,0]} \\
\chi^{(1)}_{[1,0,1]}\chi^{(2)}_{[1,0,1]} + \chi^{(1)}_{[1,0,-1]}\chi^{(2)}_{[1,0,-1]} &= \chi^{(10)}_{[3,6,1]} + \chi^{(10)}_{[7,6,1]} \\
\chi^{(1)}_{[1,0,1]}\chi^{(2)}_{[1,0,-1]} + \chi^{(1)}_{[1,0,-1]}\chi^{(2)}_{[1,0,1]} &= \chi^{(10)}_{[3,6,-1]} + \chi^{(10)}_{[7,6,-1]} \\
\chi^{(1)}_{[1,0,1]}\chi^{(2)}_{[1,2,1]} + \chi^{(1)}_{[1,0,-1]}\chi^{(2)}_{[1,2,-1]} &= \chi^{(10)}_{[3,0,1]} + \chi^{(10)}_{[7,0,1]} \\
\chi^{(1)}_{[1,0,1]}\chi^{(2)}_{[1,2,-1]} + \chi^{(1)}_{[1,0,-1]}\chi^{(2)}_{[1,2,1]} &= \chi^{(10)}_{[3,0,-1]} + \chi^{(10)}_{[7,0,-1]}
\end{aligned}$$

B. Boundary states for SU(2)

B.1 Coefficients for E_6

$$\begin{array}{rccccccc}
l = & & 0 & 3 & 4 & 6 & 7 & 10 \\
\psi^{(l)}_1 = & \Big(& \frac{1}{2}\sqrt{\frac{3-\sqrt{3}}{6}}, & \frac{1}{2}, & \frac{1}{2}\sqrt{\frac{3+\sqrt{3}}{6}}, & \frac{1}{2}\sqrt{\frac{3+\sqrt{3}}{6}}, & \frac{1}{2}, & \frac{1}{2}\sqrt{\frac{3-\sqrt{3}}{6}} \Big) \\
\psi^{(l)}_2 = & \Big(& \frac{1}{2}\sqrt{\frac{3+\sqrt{3}}{6}}, & \frac{1}{2}, & \frac{1}{2}\sqrt{\frac{3-\sqrt{3}}{6}}, & -\frac{1}{2}\sqrt{\frac{3-\sqrt{3}}{6}}, & -\frac{1}{2}, & -\frac{1}{2}\sqrt{\frac{3+\sqrt{3}}{6}} \Big) \\
\psi^{(l)}_3 = & \Big(& \frac{1}{2}\sqrt{\frac{3+\sqrt{3}}{3}}, & 0, & -\frac{1}{2}\sqrt{\frac{3-\sqrt{3}}{3}}, & -\frac{1}{2}\sqrt{\frac{3-\sqrt{3}}{3}}, & 0, & \frac{1}{2}\sqrt{\frac{3+\sqrt{3}}{3}} \Big) \\
\psi^{(l)}_4 = & \Big(& \frac{1}{2}\sqrt{\frac{3+\sqrt{3}}{6}}, & -\frac{1}{2}, & \frac{1}{2}\sqrt{\frac{3-\sqrt{3}}{6}}, & -\frac{1}{2}\sqrt{\frac{3-\sqrt{3}}{6}}, & \frac{1}{2}, & -\frac{1}{2}\sqrt{\frac{3+\sqrt{3}}{6}} \Big) \\
\psi^{(l)}_5 = & \Big(& \frac{1}{2}\sqrt{\frac{3-\sqrt{3}}{6}}, & -\frac{1}{2}, & \frac{1}{2}\sqrt{\frac{3+\sqrt{3}}{6}}, & \frac{1}{2}\sqrt{\frac{3+\sqrt{3}}{6}}, & -\frac{1}{2}, & \frac{1}{2}\sqrt{\frac{3-\sqrt{3}}{6}} \Big) \\
\psi^{(l)}_6 = & \Big(& \frac{1}{2}\sqrt{\frac{3-\sqrt{3}}{3}}, & 0, & -\frac{1}{2}\sqrt{\frac{3+\sqrt{3}}{3}}, & \frac{1}{2}\sqrt{\frac{3+\sqrt{3}}{3}}, & 0, & -\frac{1}{2}\sqrt{\frac{3-\sqrt{3}}{3}} \Big)
\end{array}$$

JHEP03(2007)038

B.2 Coefficients for E_7

$$
\begin{array}{rcccccccc}
l = & & 0 & 4 & 6 & 8 & 10 & 12 & 16 \\
\psi_1^{(l)} & = (& a, & c, & b, & \frac{1}{\sqrt{3}}, & b, & c, & a \ ) \\
\psi_2^{(l)} & = (& e, & f, & d, & 0, & -d, & -f, & -e \ ) \\
\psi_3^{(l)} & = (& c, & b, & -a, & -\frac{1}{\sqrt{3}}, & -a, & b, & c \ ) \\
\psi_4^{(l)} & = (& f, & -d, & -e, & 0, & e, & d, & -f \ ) \\
\psi_5^{(l)} & = (& \frac{1}{\sqrt{6}}, & -\frac{1}{\sqrt{6}}, & \frac{1}{\sqrt{6}}, & 0, & \frac{1}{\sqrt{6}}, & -\frac{1}{\sqrt{6}}, & \frac{1}{\sqrt{6}} \ ) \\
\psi_6^{(l)} & = (& d, & -e, & f, & 0, & -f, & e, & -d \ ) \\
\psi_7^{(l)} & = (& b, & -a, & -c, & \frac{1}{\sqrt{3}}, & -c, & -a, & b \ )
\end{array}
$$

where

$$
\begin{aligned}
a &= \left(18 + 12\sqrt{3}\cos\frac{\pi}{18}\right)^{-\frac{1}{2}}, & b &= \left(18 + 12\sqrt{3}\cos\frac{11\pi}{18}\right)^{-\frac{1}{2}}, \\
c &= \left(18 + 12\sqrt{3}\cos\frac{13\pi}{18}\right)^{-\frac{1}{2}}, & d &= \left(12(1 + \cos\frac{\pi}{9})\right)^{-\frac{1}{2}}, \\
e &= \left(12(1 + \cos\frac{5\pi}{9})\right)^{-\frac{1}{2}}, & f &= \left(12(1 + \cos\frac{7\pi}{9})\right)^{-\frac{1}{2}}.
\end{aligned}
$$

B.3 Coefficients for E_8

$$
\begin{array}{rccccccccc}
l = & & 0 & 6 & 10 & 12 & 16 & 18 & 22 & 28 \\
\psi_1^{(l)} & = (& a, & f, & c, & d, & d, & c, & f, & a \ ) \\
\psi_2^{(l)} & = (& b, & e, & h, & g, & -g, & -h, & -e, & -b \ ) \\
\psi_3^{(l)} & = (& c, & d, & -a, & -f, & -f, & -a, & d, & c \ ) \\
\psi_4^{(l)} & = (& d, & a, & -f, & -c, & c, & f, & -a, & -d \ ) \\
\psi_5^{(l)} & = (& e, & -h, & -g, & b, & b, & -g, & -h, & e \ ) \\
\psi_6^{(l)} & = (& f, & -c, & d, & -a, & a, & -d, & c, & -f \ ) \\
\psi_7^{(l)} & = (& g, & -b, & e, & -h, & -h, & e, & -b, & g \ ) \\
\psi_8^{(l)} & = (& h, & -g, & -b, & e, & -e, & b, & g, & -h \ )
\end{array}
$$

where

$$
\begin{aligned}
a &= \left[\frac{15(3+\sqrt{5}) + \sqrt{15(130+58\sqrt{5})}}{2}\right]^{-1/2}, & b &= \left[15 + \sqrt{75 - 30\sqrt{5}}\right]^{-1/2}, \\
c &= \left[\frac{15(3+\sqrt{5}) - \sqrt{15(130+58\sqrt{5})}}{2}\right]^{-1/2}, & e &= \left[15 - \sqrt{75 + 30\sqrt{5}}\right]^{-1/2}, \\
d &= \left[\frac{15(3-\sqrt{5}) - \sqrt{15(130-58\sqrt{5})}}{2}\right]^{-1/2}, & g &= \left[15 + \sqrt{75 + 30\sqrt{5}}\right]^{-1/2}, \\
f &= \left[\frac{15(3-\sqrt{5}) + \sqrt{15(130-58\sqrt{5})}}{2}\right]^{-1/2}, & h &= \left[15 - \sqrt{75 - 30\sqrt{5}}\right]^{-1/2}.
\end{aligned}
$$

C. Matrix factorisations

C.1 Matrix factorisations for E_6

The matrix factorisations for $W = x^3 + y^4$ are [10]

$$
\begin{aligned}
E_1 = J_5 &= \begin{pmatrix} x & y \\ y^3 & -x^2 \end{pmatrix} & E_5 = J_1 &= \begin{pmatrix} x^2 & y \\ y^3 & -x \end{pmatrix} \\
E_2 = J_4 &= \begin{pmatrix} x^2 & -xy & y^2 \\ y^3 & x^2 & -xy \\ -xy^2 & y^3 & x^2 \end{pmatrix} & E_4 = J_2 &= \begin{pmatrix} x & y & 0 \\ 0 & x & y \\ y^2 & 0 & x \end{pmatrix} \\
E_3 &= \begin{pmatrix} x & y^2 & 0 & 0 \\ y^2 & -x^2 & 0 & 0 \\ 0 & -xy & x^2 & y^2 \\ y & 0 & y^2 & -x \end{pmatrix} & J_3 &= \begin{pmatrix} x^2 & y^2 & 0 & 0 \\ y^2 & -x & 0 & 0 \\ 0 & -y & x & y^2 \\ xy & 0 & y^2 & -x^2 \end{pmatrix} \\
E_6 &= \begin{pmatrix} x & y^2 \\ y^2 & -x^2 \end{pmatrix} & J_6 &= \begin{pmatrix} x^2 & y^2 \\ y^2 & -x \end{pmatrix}
\end{aligned}
$$

The matrix factorisations for $W = x^3 + y^4 + z^2$ are [11]

$$
\begin{aligned}
E_1 = J_5 &= \begin{pmatrix} -y^2 + iz & x \\ x^2 & y^2 + iz \end{pmatrix} & J_1 = E_5 &= \begin{pmatrix} -y^2 - iz & x \\ x^2 & y^2 - iz \end{pmatrix} \\
E_2 = J_4 &= \begin{pmatrix} -y^2 + iz & 0 & xy & x \\ -xy & y^2 + iz & x^2 & 0 \\ 0 & x & iz & y \\ x^2 & -xy & y^3 & iz \end{pmatrix} & E_4 = J_2 &= \begin{pmatrix} -y^2 - iz & 0 & xy & x \\ -xy & y^2 - iz & x^2 & 0 \\ 0 & x & -iz & y \\ x^2 & -xy & y^3 & -iz \end{pmatrix} \\
E_3 &= \begin{pmatrix} -iz & -y^2 & xy & 0 & x^2 & 0 \\ -y^2 & -iz & 0 & 0 & 0 & x \\ 0 & 0 & -iz & -x & 0 & y \\ 0 & xy & -x^2 & -iz & y^3 & 0 \\ x & 0 & 0 & y & -iz & 0 \\ 0 & x^2 & y^3 & 0 & xy^2 & -iz \end{pmatrix} & J_3 &= \begin{pmatrix} iz & -y^2 & xy & 0 & x^2 & 0 \\ -y^2 & iz & 0 & 0 & 0 & x \\ 0 & 0 & iz & -x & 0 & y \\ 0 & xy & -x^2 & iz & y^3 & 0 \\ x & 0 & 0 & y & iz & 0 \\ 0 & x^2 & y^3 & 0 & xy^2 & iz \end{pmatrix} \\
J_6 = E_6 &= \begin{pmatrix} -z & 0 & x^2 & y^3 \\ 0 & -z & y & -x \\ x & y^3 & z & 0 \\ y & -x^2 & 0 & z \end{pmatrix}
\end{aligned}
$$

C.2 Matrix factorisations for E_7

For $W = x^3 + xy^3$, the matrix factorisations are given by [10]

$$
\begin{aligned}
E_1 &= x & J_1 &= x^2 + y^3 \\
E_2 &= \begin{pmatrix} x^2 & y^2 \\ xy & -x \end{pmatrix} & J_2 &= \begin{pmatrix} x & y^2 \\ xy & -x^2 \end{pmatrix} \\
E_3 &= \begin{pmatrix} x^2 & -y^2 & -xy \\ xy & x & -y^2 \\ xy^2 & xy & x^2 \end{pmatrix} & J_3 &= \begin{pmatrix} x & 0 & y \\ -xy & x^2 & 0 \\ 0 & -xy & x \end{pmatrix} \\
E_4 &= \begin{pmatrix} x & y & -y & 0 \\ y^2 & -x & 0 & -y \\ 0 & 0 & x^2 & xy \\ 0 & 0 & xy^2 & -x^2 \end{pmatrix} & J_4 &= \begin{pmatrix} x^2 & xy & y & 0 \\ xy^2 & -x^2 & 0 & y \\ 0 & 0 & x & y \\ 0 & 0 & y^2 & -x \end{pmatrix} \\
E_5 &= \begin{pmatrix} y & 0 & x \\ -x & xy & 0 \\ 0 & -x & y \end{pmatrix} & J_5 &= \begin{pmatrix} xy^2 & -x^2 & -x^2y \\ xy & y^2 & -x^2 \\ x^2 & xy & xy^2 \end{pmatrix} \\
E_6 &= \begin{pmatrix} x^2 & y \\ xy^2 & -x \end{pmatrix} & J_6 &= \begin{pmatrix} x & y \\ xy^2 & -x^2 \end{pmatrix} \\
E_7 &= \begin{pmatrix} x^2 & xy \\ xy^2 & -x^2 \end{pmatrix} & J_7 &= \begin{pmatrix} x & y \\ y^2 & -x \end{pmatrix}
\end{aligned}
$$

The other factorisations $\bar{Q}_i$ correspond to their antibranes and are given by $\bar{E}_i = J_i$, $\bar{J}_i = E_i$.

For $W = x^3 + xy^3 + z^2$, the factorisations are constructed out of the above by

$$
\hat{E}_i = \hat{J}_i = \begin{pmatrix} z\mathbf{1} & J_i \\ E_i & -z\mathbf{1} \end{pmatrix} ,
$$

so that $\hat{Q}_i$ is equal to its own antibrane.

C.3 Matrix factorisations for E_8

For $W = x^3 + y^5$ the matrix factorisations are given by [10]

$$
\begin{aligned}
E_1 &= \begin{pmatrix} x^2 & y \\ y^4 & -x \end{pmatrix} & J_1 &= \begin{pmatrix} x & y \\ y^4 & -x^2 \end{pmatrix} \\
E_2 &= \begin{pmatrix} y^4 & xy^3 & x^2 \\ -x^2 & y^4 & xy \\ -xy & -x^2 & y^2 \end{pmatrix} & J_2 &= \begin{pmatrix} y & -x & 0 \\ 0 & y & -x \\ x & 0 & y^3 \end{pmatrix} \\
E_3 &= \begin{pmatrix} 0 & x^2 & -y^3 & 0 \\ -x^2 & xy & 0 & -y^3 \\ 0 & -y^2 & -x & 0 \\ y^2 & 0 & y & -x \end{pmatrix} & J_3 &= \begin{pmatrix} y & -x & 0 & y^3 \\ x & 0 & -y^3 & 0 \\ -y^2 & 0 & -x^2 & 0 \\ 0 & -y^2 & -xy & -x^2 \end{pmatrix}
\end{aligned}
$$

$$E_4 = \begin{pmatrix} y & -x & 0 & 0 & 0 \\ x & 0 & 0 & y^2 & 0 \\ -y^2 & 0 & -x^2 & 0 & -y^3 \\ 0 & -y^2 & 0 & x & 0 \\ 0 & 0 & y^2 & y & -x \end{pmatrix} \qquad J_4 = \begin{pmatrix} y^4 & x^2 & 0 & -xy^2 & 0 \\ -x^2 & xy & 0 & -y^3 & 0 \\ 0 & -y^2 & -x & 0 & y^3 \\ -xy^2 & y^3 & 0 & x^2 & 0 \\ -y^3 & 0 & -y^2 & xy & -x^2 \end{pmatrix}$$

$$E_5 = \begin{pmatrix} y^4 & xy^2 & x^2 & 0 & 0 & xy \\ -x^2 & y^3 & xy & -x & 0 & 0 \\ -xy^2 & -x^2 & y^3 & 0 & -xy & 0 \\ 0 & 0 & 0 & y & -x & 0 \\ 0 & 0 & 0 & 0 & y^2 & -x \\ 0 & 0 & 0 & x & 0 & y^2 \end{pmatrix} \qquad J_5 = \begin{pmatrix} y & -x & 0 & 0 & 0-x & \\ 0 & y^2 & -x & xy & 0 & 0 \\ x & 0 & y^2 & 0 & xy & 0 \\ 0 & 0 & 0 & y^4 & xy^2 & x^2 \\ 0 & 0 & 0 & -x^2 & y^3 & xy \\ 0 & 0 & 0 & -xy^2 & -x^2 & y^3 \end{pmatrix}$$

$$E_6 = \begin{pmatrix} x^2 & y^2 & 0 & xy \\ y^3 & -x & -y^2 & 0 \\ 0 & 0 & x & y^2 \\ 0 & 0 & y^3 & -x^2 \end{pmatrix} \qquad J_6 = \begin{pmatrix} x & y^2 & 0 & y \\ y^3 & -x^2 & -xy^2 & 0 \\ 0 & 0 & x^2 & y^2 \\ 0 & 0 & y^3 & -x \end{pmatrix}$$

$$E_7 = \begin{pmatrix} x & y^2 \\ y^3 & -x^2 \end{pmatrix} \qquad J_7 = \begin{pmatrix} x^2 & y^2 \\ y^3 & -x \end{pmatrix}$$

$$E_8 = \begin{pmatrix} y^4 & xy^2 & x^2 \\ -x^2 & y^3 & xy \\ -xy^2 & -x^2 & y^3 \end{pmatrix} \qquad J_8 = \begin{pmatrix} y & -x & 0 \\ 0 & y^2 & -x \\ x & 0 & y^2 \end{pmatrix}$$

and their respective antibranes.

The factorisations for $W = x^3 + y^5 + z^2$ are constructed in the same way as for E_7.

References

[1] D. Gepner, *Space-time supersymmetry in compactified string theory and superconformal models*, *Nucl. Phys.* **B 296** (1988) 757.

[2] A. Kapustin and Y. Li, *D-branes in Landau-Ginzburg models and algebraic geometry*, *JHEP* **12** (2003) 005 [hep-th/0210296].

[3] I. Brunner, M. Herbst, W. Lerche and B. Scheuner, *Landau-Ginzburg realization of open string TFT*, *JHEP* **11** (2006) 043 [hep-th/0305133].

[4] A. Kapustin and Y. Li, *Topological correlators in Landau-Ginzburg models with boundaries*, *Adv. Theor. Math. Phys.* **7** (2004) 727 [hep-th/0305136].

[5] A. Kapustin and Y. Li, *D-branes in topological minimal models: the Landau-Ginzburg approach*, *JHEP* **07** (2004) 045 [hep-th/0306001].

[6] C.I. Lazaroiu, *On the boundary coupling of topological Landau-Ginzburg models*, *JHEP* **05** (2005) 037 [hep-th/0312286].

[7] M. Herbst and C.-I. Lazaroiu, *Localization and traces in open-closed topological Landau-Ginzburg models*, *JHEP* **05** (2005) 044 [hep-th/0404184].

[8] K. Hori and J. Walcher, *F-term equations near Gepner points*, *JHEP* **01** (2005) 008 [hep-th/0404196].

JHEP03(2007)038

[9] I. Brunner and M.R. Gaberdiel, *The matrix factorisations of the D-model*, *J. Phys.* **A 38** (2005) 7901 [hep-th/0506208].

[10] Y. Yoshino, *Cohen-Macaulay modules over Cohen-Macaulay rings*, London Mathematical Society Lecture Note Series **146**, Cambridge University Press (1990).

[11] H. Kajiura, K. Saito and A. Takahashi, *Matrix factorizations and representations of quivers II: type ADE case*, math.AG/0511155.

[12] R.E. Behrend, P.A. Pearce, V.B. Petkova and J.-B. Zuber, *Boundary conditions in rational conformal field theories*, *Nucl. Phys.* **B 570** (2000) 525 [hep-th/9908036] [Erratum *ibid.* **579** (2000) 707].

[13] M. Naka and M. Nozaki, *Boundary states in Gepner models*, *JHEP* **05** (2000) 027 [hep-th/0001037].

[14] W. Lerche, C.A. Lutken and C. Schweigert, *D-branes on ALE spaces and the ADE classification of conformal field theories*, *Nucl. Phys.* **B 622** (2002) 269 [hep-th/0006247].

[15] C.I. Lazaroiu, *String field theory and brane superpotentials*, *JHEP* **10** (2001) 018 [hep-th/0107162].

[16] M. Herbst, C.-I. Lazaroiu and W. Lerche, *D-brane effective action and tachyon condensation in topological minimal models*, *JHEP* **03** (2005) 078 [hep-th/0405138].

[17] M. Herbst, C.-I. Lazaroiu and W. Lerche, *Superpotentials, A(infinity) relations and WDVV equations for open topological strings*, *JHEP* **02** (2005) 071 [hep-th/0402110].

[18] J. Knapp and H. Omer, *Matrix factorizations, minimal models and massey products*, *JHEP* **05** (2006) 064 [hep-th/0604189].

[19] D. Gepner, *Lectures on $N = 2$ string theory*, PUPT-1121, Lectures at Spring School on Superstrings, Trieste, Italy, Apr 3-14 (1989).

[20] T. Gannon, $\mathrm{U}(1)$*-m modular invariants, $N = 2$ minimal models and the quantum Hall effect*, *Nucl. Phys.* **B 491** (1997) 659 [hep-th/9608063].

[21] M. Herbst, W. Lerche and D. Nemeschansky, *Instanton geometry and quantum A(infinity) structure on the elliptic curve*, hep-th/0603085.

[22] S.K. Ashok, E. Dell'Aquila, D.-E. Diaconescu and B. Florea, *Obstructed D-branes in Landau-Ginzburg orbifolds*, *Adv. Theor. Math. Phys.* **8** (2004) 427 [hep-th/0404167].

[23] J. Polchinski, *String theory. Vol. 1: an introduction to the bosonic string*, Cambridge Monographs on Mathematical Physics, Cambridge University Press (1998).

[24] N.P. Warner, *$N = 2$ supersymmetric integrable models and topological field theories*, hep-th/9301088.

[25] J.M. Maldacena, G.W. Moore and N. Seiberg, *Geometrical interpretation of D-branes in gauged WZW models*, *JHEP* **07** (2001) 046 [hep-th/0105038].